

**EXTENDING THE STANDARD MODEL:  
AN UPPER BOUND FOR A NEUTRINO MASS FROM THE RARE DECAY  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$**

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**Abstract:** The standard model seeming at a loss to account for the present experimental average rate for the rare decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ , I tackle the question with the extension of the Glashow-Salam-Weinberg model to an  $SU(2)_L \times U(1)$  gauge theory of  $J = 0$  mesons proposed in [7], in which, in addition, the neutrinos are given Dirac masses from Yukawa couplings to the Higgs boson. The latter triggers a new contribution to this decay through flavor changing neutral currents that arise in the quartic term of the symmetry breaking potential; it becomes sizeable for a neutrino mass in the  $MeV$  range; the experimental upper limit for the decay rate translates into an upper bound of  $5.5 MeV$  for the mass of the neutrino, three times lower than present direct bounds.

**PACS:** 12.60.Fr    13.20.Eb    14.60.Pq



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# 1 Introduction

The absence of flavor changing neutral currents at tree level is one of the most severe criteria to select among the various candidates which could generalize the Glashow-Salam-Weinberg model of electroweak interactions. They are in general linked to higher order corrections and their experimental quest is of high interest for the pursuit of theoretical investigations and the search for hints of new physics beyond the standard model.

Among the decays of particular interest involving flavor changing neutral currents are

$$K^+ \longrightarrow \pi^+ \ell \bar{\ell} \quad (1)$$

where the  $\ell$ 's stand for leptons which can be charged (electrons or muons) or neutral (neutrinos).

One event compatible with a two-neutrinos final state has recently been observed [1] corresponding to a branching ratio

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{exp} = 4.2^{+9.7}_{-3.5} 10^{-10}, \quad (2)$$

while recent theoretical calculations [2] find, for massless neutrinos, an absolute upper bound

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{th} < 1.22 \cdot 10^{-10}, \quad (3)$$

and the authors of [2] claim “*a clear conflict with the Standard Model if  $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$  should be measured at  $2 \cdot 10^{-10}$* ”, that is less than one-half the experimental average (2).

At the same time, we are witnessing a dramatic change in our perception of neutrinos since there seems to be more and more compelling evidence [3] that their flavor eigenstates oscillate [4] during their travel across vacuum or matter, which is the best model-independent sign of them being massive.

The question that can naturally be raised is whether massive neutrinos can substantially increase the rate of the above decay in the standard framework, specially through penguin-like diagrams where the Higgs boson couples to the internal  $W$  gauge boson or to the very massive top quark. The computations have just been performed [5] in the case of  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  and showed that the influence of massive neutrinos is totally negligible when mass and flavour leptonic eigenstates coincide, and has a relative upper bound of no more than 1/10 when flavour mixing is allowed.

Rare semi-leptonic  $K$  decays provide consequently a good testing ground for physics beyond the standard model [6].

I investigate below the influence of neutrino masses on the decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  in the framework of the extension of the Glashow-Salam-Weinberg model to an  $SU(2)_L \times U(1)$  gauge theory of  $J = 0$  mesons proposed in [7].

It is built with a maximum compatibility with the standard model in the quark-gauge sector <sup>1</sup> : chiral and electroweak properties of quarks are included from the start <sup>2</sup> ; it however differs in the Higgs-scalar sector in that the three Goldstones of the broken symmetry(es) are now related [9] through a scaling factor (see (6) below) to pseudoscalar mesons and the Higgs boson is naturally incorporated as one among the  $J = 0$  mesons, which all are considered to transform like quark-antiquark composite fields. The  $SU(2)_L \times U(1)$  electroweak group is embedded into the larger chiral  $U(N)_L \times U(N)_R$  group ( $N/2$  is the number of generations), which is only possible when  $N$  is even; this is to be compared with chiral perturbation theory [8], which is always performed for an odd (three) number of flavours; in the present framework, the relevant chiral breaking appears instead to be the one of  $SU(2)_L \times SU(2)_R$  into its diagonal subgroup, the custodial  $SU(2)$ . The process under concern can accordingly, now, be also mediated by the Higgs boson which, because of the Cabibbo-Kobayashi-Maskawa (CKM) rotation, connects, on one side through the mexican-hat potential, pseudoscalar mesons with different flavors <sup>3</sup> , and, on the other side couples through Yukawa couplings to massive neutrinos.

<sup>1</sup>Relying on this, the contributions involving gauge bosons will be assumed to be of the same order of magnitude as when computed in the standard model.

<sup>2</sup>The  $SU(2)_L \times U(1)$  electroweak group is embedded into the larger chiral  $U(N)_L \times U(N)_R$  group ( $N/2$  is the number of generations), which is only possible when  $N$  is even; this is to be compared with chiral perturbation theory [8], which is always performed for an odd (three) number of flavours; in the present framework, the relevant chiral breaking appears instead to be the one of  $SU(2)_L \times SU(2)_R$  into its diagonal subgroup, the custodial  $SU(2)$ .

<sup>3</sup>The occurrence of these flavor changing neutral currents have already been mentioned in [9] in the decays of the  $Z$  boson into two leptons and two pseudoscalar mesons.

## 2 The (non-standard) Higgs contribution to $K^+ \rightarrow \pi^+ \ell \bar{\ell}$

### 2.1 Theoretical framework

The theoretical framework has been set in [7] and [9] (section 2); we work in the approximation where the electroweak quadruplet of  $J = 0$  mesons containing the Higgs boson and the three (pseudoscalar) goldstone bosons is <sup>4</sup>

$$(H, \vec{G}) = (\mathbb{S}^0, \vec{\mathbb{P}})(\mathbb{D}_1); \quad (4)$$

this is akin to taking as non-vanishing only the flavor-diagonal quark condensates and to choosing all of them to be identical <sup>5</sup>.

The quartic term in the “mexican hat” potential triggers, after symmetry breaking, a coupling between the Higgs boson and two of the three Goldstones of the broken electroweak symmetry (or, equivalently, of the breaking of  $SU(2)_L \times SU(2)_R$  into the custodial  $SU(2)_V$  [7]). Because of the CKM rotation, the Goldstones are not flavor eigenstates but mixtures of them, which entails new type of flavour changing neutral currents, connecting in particular  $K^+$  and  $\pi^+$  mesons.

On the other side, the Yukawa couplings that are introduced between leptons and the real quadruplet (complex doublet) (4) to give Dirac masses to the former couple the Higgs boson to leptonic mass eigenstates (which may not be flavour eigenstates in the case of neutrinos).

So, working with two generations ( $N = 4$ ), the two vertices involved in the diagram of Fig. 1 write

$$\begin{aligned} V_{hK\pi} &= -i\sqrt{2}c_\theta s_\theta \lambda v, \\ V_{h\ell\bar{\ell}} &= i\frac{m_\ell^D}{v/\sqrt{2}}, \end{aligned} \quad (5)$$

where  $\lambda$  is the coupling constant of the quartic term in the mexican-hat potential,  $c_\theta$  and  $s_\theta$  are the cosine and sine of the Cabibbo angle,  $v/\sqrt{2}$  is the vacuum expectation value of the Higgs boson, and  $m_\ell^D$  is the Dirac mass of the outgoing lepton. The mass of the Higgs boson is  $M_h^2 = \lambda v^2$ ; as it is supposed to be much larger than the mass of the incoming mesons, we shall neglect the momentum dependence of the Higgs propagator, which makes the amplitude independent of  $\lambda$ .

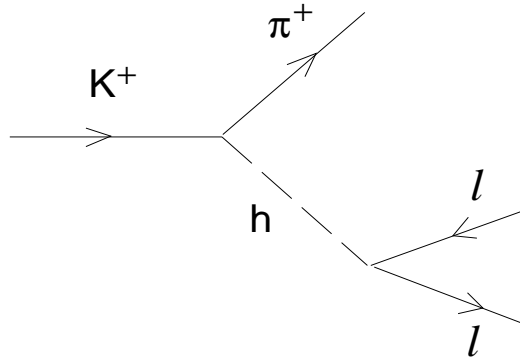


Fig. 1: The Higgs mediated decay  $K^+ \rightarrow \pi^+ \ell \bar{\ell}$

<sup>4</sup>The  $N^2/2$  quadruplets of  $J = 0$  mesons corresponding to  $N$  flavors of quarks are labeled by  $(N/2)^2$  real matrices  $\mathbb{D}$  of dimension  $N/2 \times N/2$  and  $\mathbb{D}_1$  is the corresponding unit matrix; see [9] for the notations.

<sup>5</sup>In this limit, the two neutral kaons are not coupled to the Higgs boson, which consequently does not participate, in particular, to their decays into  $\pi^0 \ell \bar{\ell}$ .

## 2.2 Calculation of the decay rate for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The calculation includes the normalization factor [9]

$$\mathbf{b} = \frac{\langle H \rangle}{2f_0} = \frac{v}{2\sqrt{2}f_0} \quad (6)$$

which relates the fields of dimension  $[mass]$  in the Lagrangian to observed asymptotic mesons;  $f_0$  is the generic leptonic decay constant of pseudoscalar mesons, that we consider to be the same for  $K$  and  $\pi$ ;  $\mathbf{b}$  modifies accordingly (see [9]) the phase-space measure for the outgoing pion, which becomes itself proportional to  $\mathbf{b}^2$ ; this makes the decay rate (7) below proportional to  $1/\mathbf{b}^2$ , that is to  $f_0^2$ , and yields a  $G_F^3$  dependence though one deals with a tree-level diagram.

The decay rate writes

$$\Gamma_{K^+ \rightarrow \pi^+ \nu \bar{\nu}} = (s_\theta c_\theta)^2 \frac{\sqrt{2}f_0^2 G_F^3 (m_\nu^D)^2}{\pi^3 M_K^3} \left[ \frac{(M_K^2 - M_\pi^2)^3}{3} + \frac{M_K^2 + M_\pi^2}{2} \left( M_K^4 - M_\pi^4 - 2M_K^2 M_\pi^2 \ln \frac{M_K^2}{M_\pi^2} \right) \right], \quad (7)$$

where having neglected their masses in the phase-space integration for neutrinos enabled to get an analytic expression, in which the sole dependence on the Dirac neutrino mass  $m_\nu^D$  comes from their coupling to the Higgs boson.

The value of the corresponding branching ratio is plotted on Fig. 2 as a function of  $m_\nu^D$ .

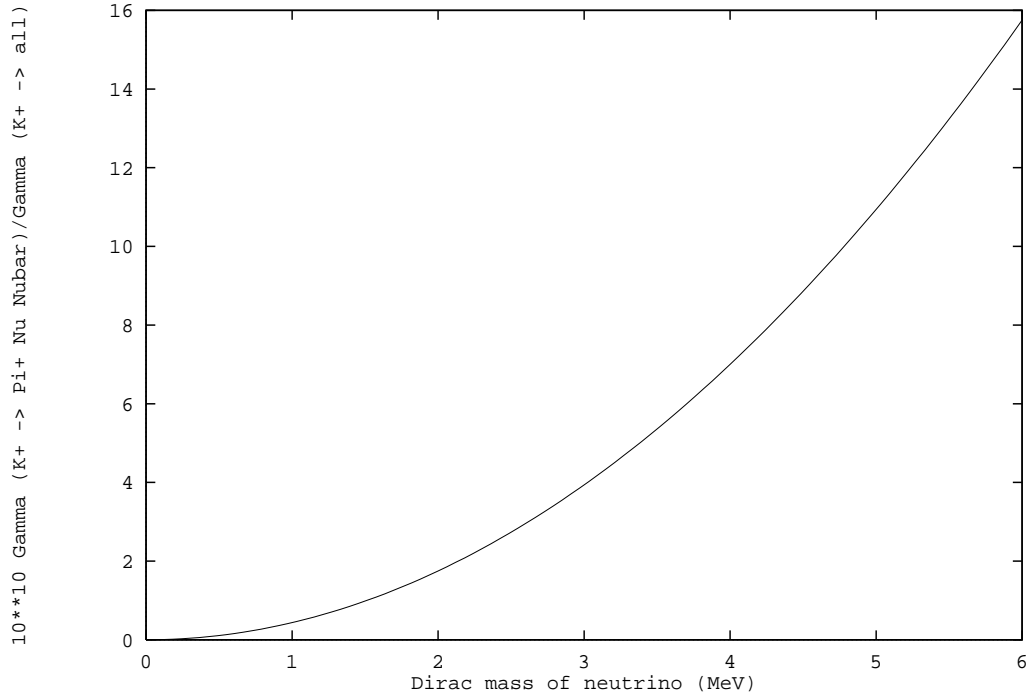


Fig. 2: The branching ratio  $\Gamma_{K^+ \rightarrow \pi^+ \nu \bar{\nu}} / \Gamma_{K^+ \rightarrow \text{all}}$  as a function of  $m_\nu^D$

## 2.3 Influence of the neutrino spectrum

We have supposed a hierarchical scheme for the neutrino masses and only considered the coupling of the Higgs to the heaviest one.

In case non-sterile neutrinos are roughly degenerate, the three corresponding, nearly equivalent, amplitudes should be added; as a result, Fig. 2 should be read for  $3 m_\nu^D$  instead of  $m_\nu^D$ .

As detecting and identifying the outgoing neutrinos by their flavor properties is well beyond present experimental ability, there is no purpose in introducing the leptonic mixing matrix and studying a precise channel.

## 2.4 Calculation of the decay rates for $K^+ \rightarrow \pi^+ e^+ e^-$ and $K^+ \rightarrow \pi^+ \mu^+ \mu^-$

It is important to check that the same mechanism does not grossly alters the standard predictions in the cases where the two outgoing leptons are charged (electrons or muons). The calculations go along the same way, except that one has to keep the dependence on their masses when performing the phase space integral; the final evaluation can then only be numerical.

One gets

$$\Gamma_{K^+ \rightarrow \pi^+ e^+ e^-} = 6.4 \cdot 10^{-29} \text{ GeV} \quad (8)$$

and

$$\Gamma_{K^+ \rightarrow \pi^+ \mu^+ \mu^-} = 8.37 \cdot 10^{-25} \text{ GeV}; \quad (9)$$

this corresponds to the branching ratios

$$BR(K^+ \rightarrow \pi^+ e^+ e^-) = 1.19 \cdot 10^{-12} \quad (10)$$

and

$$BR(K^+ \rightarrow \pi^+ \mu^+ \mu^-) = 1.56 \cdot 10^{-8}, \quad (11)$$

to be compared with the experimental values [10]

$$BR_{exp}(K^+ \rightarrow \pi^+ e^+ e^-) = 2.74 \pm .23 \cdot 10^{-7} \quad (12)$$

and

$$BR_{exp}(K^+ \rightarrow \pi^+ \mu^+ \mu^-) = 5 \pm 1 \cdot 10^{-8}. \quad (13)$$

This shows that the non-standard Higgs contribution is negligible in the case of two outgoing electrons, and within the range of the experimental uncertainty in the case of two outgoing muons.

In those two last cases, ours is consequently not expected to modify present theoretical calculations.

## 3 An upper bound for the Dirac mass term of the heaviest non-sterile neutrino

We suppose that standard calculations could at the maximum account for a branching ratio  $BR_{K^+ \rightarrow \pi^+ \nu \bar{\nu}} \approx 1.5 \cdot 10^{-10}$ ; having no information on the relative sign between the standard amplitude and the new Higgs-mediated contribution, we can only say that the latter dominates when it yields a partial decay rate at least twice as large as the limit above, that is for  $m_\nu^D \geq 2.5 \text{ MeV}$ .

Then the experimental upper bound (2) entails

$$m_\nu^D \leq 5.5 \text{ MeV} \quad (14)$$

which is three times lower than the direct bound [10]

$$m_{\nu_\tau} \leq 18.2 \text{ MeV}. \quad (15)$$

The average experimental value (2) corresponds to

$$m_\nu^D \approx 3 \text{ MeV}. \quad (16)$$

This value is much higher than generally presumed order of magnitudes for masses of non-sterile neutrinos coming from recent results on solar and atmospheric neutrinos [3], setting the scales for the mass splittings,

combined with absolute upper bounds on neutrinos masses [11], in particular the ones coming from studying the spectrum of the  $\beta$  decay of  ${}^3H$  [12]

$$m_{\nu_e} \leq 3.9 \text{ eV} \text{ (90\% CL)}.$$

The only known mechanism which could account for such a discrepancy between the observed neutrino mass and a Dirac mass term is the so-called “see-saw” mechanism [13] in which, in addition to the Dirac mass term, a Majorana mass term  $\mathcal{M} \gg m_\nu^D$  is generated through a coupling to a new triplet of scalars; it is associated with a new scale of physics (right-handed gauge fields, grand unified theories *etc*). It is thus of interest to study this case here. However, as shown below, the only effect of advocating for such a mechanism is to replace in the expression of the decay rate (7) the Dirac neutrino mass  $m_\nu^D$  by the one of the lightest Majorana eigenstate. The conclusion is thus maintained that the new process advocated here has to be considered only for neutrino masses in the  $MeV$  range, and is negligible if they lie in the  $eV$  range.

After the diagonalisation of this general mass matrix involving the two types of mass terms [3], the two eigenstates are Majorana neutrinos  $\nu_1 = i(\nu_L - (\nu_L)^c)$ ,  $\nu_2 = \nu_R + (\nu_R)^c$ , with masses respectively

$$m_1 = (m_\nu^D)^2/\mathcal{M}, \quad m_2 \approx \mathcal{M}. \quad (17)$$

But, while the left-handed neutrino is mostly made of the lightest eigenstate  $\nu_1$ , the right-handed one which, in the process under scrutiny, is coupled through the Higgs boson to the left-handed neutrino, is mostly made of the heaviest one  $\nu_2$ , which one does not expect to be produced here, plus only a very small admixture of the light one, in the proportion  $m_\nu^D/m_2 \approx m_\nu^D/\mathcal{M}$ .

The decay rate (7) has thus now to be multiplied by  $(m_\nu^D/\mathcal{M})^2 \approx (m_1/m_\nu^D)^2$ , where we have used (17). This has the global effect of replacing in (7) the factor  $(m_\nu^D)^2$  by the light neutrino mass  $m_1^2$ .

## 4 Conclusion

By providing a unified view of  $J = 0$  mesons which includes the Higgs boson, the extension [7] of the electroweak standard model enables, in this sector, predictions which depart from the Glashow-Salam-Weinberg model.

After  $K \rightarrow \pi\pi$  decays [14] and the disintegrations of the  $Z$  boson into two pseudoscalar mesons and two leptons [9], we have extended here our investigations to the rare semi-leptonic decays of kaons; we have shown that, in this framework and unlike in the standard model, decay rates for  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  in agreement with present experimental bounds can be accounted for with neutrino masses in the  $MeV$  range, which is not yet excluded experimentally.

*Acknowledgments:* It is a pleasure to thank S.T. Petcov and X.Y. Pham for suggestions, comments and advice, and also the referee for pointing out a misinterpretation in the first version of this work.

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